Stability Analysis of Single Prediction Horizon Continuous-Control-Set Model-Predictive-Control of Buck Converter with Voltage and Current Control

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***Abstract*—This paper focuses on implementing Model Predictive Control (MPC) for buck converters. MPC has been increasingly adopted due to its superior dynamic performance in controlling linear and non-linear plants compared to classical frequency domain controllers. However, ensuring stability while maintaining high response speeds presents a significant challenge to the control designer. Specifically, for higher order systems with more than one state variable, attempting to increase the speed of response solely by considering the output variable can lead to limit cycle oscillations. This phenomenon has been reported for buck converters where inductor current can have limit cycle oscillation, when the output voltage is controlled.**

To address this challenge, this paper proposes a continuous control set MPC (CCS-MPC) algorithm with a cost function considering both state variables. This novel control algorithm uses a weighing factor to control both state variables, inductor current and capacitor voltage. This weighing factor is determined based on stability analysis, and its variation is also investigated. The proposed solution provides superior dynamic performance compared to classical controllers and superior dynamic and steady state performance compared to dead beat voltage control of buck converter with single prediction horizon. Simulation and experimental setups are used to verify the proposed algorithm's effectiveness.

***Index Terms*—Stability analysis, Continuous control set model predictive control, CCS-MPC, dead beat control, Buck converter.**

I. Introduction

The frequency domain controller for the buck converter are tuned to have a bandwidth much lower than the switching frequency, [1]. Some emerging control strategies can improve the transient response with constant and variable switching frequency control, [2]. Fixed switching frequency control can be implemented using a microcontroller, and the variable switching frequency control requires additional analog circuits. This makes constant switching frequency control preferable in most applications, and the scope of this paper is limited to them.

Model predictive control (MPC), one such algorithm, is being adopted in power electronic converters due to their fast transient performance. MPC is further classified as continuous control set (CCS) MPC and finite control set (FCS) MPC, based on the control variable set. FCS MPC computes the input from a finite element set that minimizes the cost function. This is often done by computing the cost function for all the elements of the control set and selecting the input with the least value for the cost function. CCS MPC computes the input from a continuous set with infinite elements. The cost function is minimized analytically to compute the optimal input. This computation is challenging for non-linear systems as the closed-form expression for the input may not exist. Therefore, usually, a single prediction horizon is preferred for implementing MPC in power converters.

A few modern constant switching frequency control techniques for buck converters are compared in [3]; the first technique minimizes the cost function involving an error in state variables and a change in input. The optimal input is computed by making the first derivative of the cost function equal to zero; a non-linear equation is solved by one iteration of the Newton algorithm. The strategy limits the inductor current and uses a load observer to sense the load change. The cost function involving inductor current and the output voltage is minimized in [4] with added Kalman filter to take care of offset error. The closed-form solution of optimal input is obtained in [5] by minimizing the cost function involving error and difference in input from steady-state. The strategy is valid for constant power loads with a higher-order sliding mode observer to estimate load. The paper focuses on higher-order sliding mode observer to have an offset-free operation. Disturbance observer to eliminate offset is also discussed in [6], [7]. MPC for parameter varying buck converter is discussed in [8].

One major challenge in the control algorithm with a faster transient response is to ensure stability for the higher-order system since the optimal input is computed considering the dynamics only for a single cycle. The buck converter is a second-order system that exhibits instability for single predictive horizon CCS-MPC voltage control, [9]. This problem is analytically estimated, and a novel control strategy is proposed

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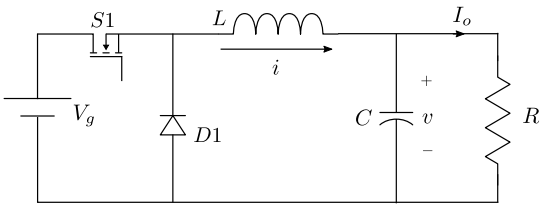


Fig. 1: Schematic of a buck converter

in this paper with a superior transient response. The rest of the paper is organized as follows. The second section describes the proposed CCS-MPC algorithm for buck converter, and stability analysis is provided in the third section. The fourth section provides the results, and the fifth section concludes the paper.

II. Proposed Model Predictive Control Algorithm

Sampled data model (SDM) of buck converter given in Fig. 1 from [10] is used for its dynamic modeling. This discrete non-linear model of the plant expresses the state variables, inductor current, and capacitor voltage in the next cycle as a function of state variables in the current cycle and input (duty ratio) in the current cycle, (1). The terms A, B, C, D, E, and F are functions of T_s (switching time period), L (filter inductance), C (filter capacitance), and R (load resistance). The converter parameters are per unitized for generalizing the analysis with the variables $\omega = T_s/\sqrt{LC}$ and $\zeta = (\sqrt{L/C})/(2R)$.

$$\begin{bmatrix} i[k+1] \\ v[k+1] \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} i[k] \\ v[k] \end{bmatrix} + \begin{bmatrix} E(d_k) \\ F(d_k) \end{bmatrix} V_g \tag{1}$$

A. Deadbeat voltage control of buck converter

One cycle control of output voltage was implemented in [10] with a cost function (2), error in output voltage minimized for a single predictive horizon. Single prediction horizon model predictive voltage control minimizes the error in output voltage that is equivalent to dead beat voltage control. v^* is the reference voltage and $v[k+2]$ is the voltage in $(k+2)^{th}$ cycle. The minimization happens in the $(k+2)^{th}$ cycle due to one cycle delay for the finite computation time. This delay is compensated by estimating the state variables in $(k+1)^{th}$ cycle using the SDM. Further duty ratio in $(k+1)^{th}$ cycle, d_{k+1} is computed in k^{th} cycle that minimize the error in $(k+2)^{th}$ cycle. This has a better transient response to the classical frequency domain controller in terms of rise time and settling time.

$$J = (v^* - v[k+2])^2 \tag{2}$$

However, it has been shown that the converter is not stable for the entire range of duty ratio in [9]. The critical duty ratio, which determines the stability limit, was analytically obtained. Beyond this critical duty ratio, the algorithm fails to operate at a constant duty ratio and shows oscillation around the required steady state operating point. This is reflected in inductor current and output voltage, referred as steady state instability or limit cycle oscillation, shown in Fig. 2.

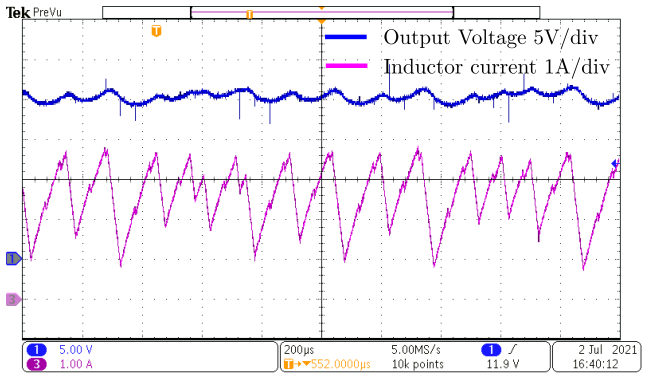


Fig. 2: Limit cycle oscillations for Single predictive horizon model predictive voltage control (Dead beat voltage control) of buck converter at higher duty ratio ($V_g = 30V$, $v_{ref} = 20V$).

In addition to steady state instability, transient stability is also poor with the output voltage exhibiting large overshoots for a step change in reference. This is highlighted in Fig. 9b though the initial and final operating point are within stable limit. Therefore, the algorithm must be modified to eliminate the limit cycle oscillation in inductor current for a higher duty ratio and improve the transient performance.

B. Proposed algorithm

A modified cost function is proposed in this paper, (3), that considers both the state variables inductor current and capacitor voltage. Considering inductor current in the cost function improves the stability region which leads to stable strategy over the entire region of operation. The algorithm compensates for the one-cycle delay due to the computation time and duty cycle updation in the next cycle [11]. The representative diagram for the control algorithm is given in Fig. 3. This includes two steps delay compensation and cost function minimization. At the start of k^{th} cycle, output voltage v_k and inductor current i_k are sensed. Due to finite computation time taken, duty ratio in k^{th} cycle d_k cannot be computed as it cannot be implemented in current cycle. Therefore, in order to compute duty ratio in next cycle d_{k+1} state variable at the start of $(k+1)^{th}$ cycle is estimated, denoted as v^e_{k+1} and i^e_{k+1}, using (1). This step is delay compensation.

In the next step, using estimated state variable at the start of $(k+1)^{th}$ cycle, duty cycle for the $(k+1)^{th}$ cycle d_{k+1} is computed. This is done by minimizing the cost function as given in (3).

$$J = \{(\alpha(v^* - v[k+2]) + (1-\alpha)(i^* - i[k+2])\}^2 \tag{3}$$

The minimization is cost function is achieved by making $J = 0$. Exponential and trigonometric function involving the duty ratio is approximated with polynomial approximation to get an analytical expression for d_{k+1}. Polynomial approximation is valid for all practically designed converter where the LC corner frequency is smaller than the switching frequency.

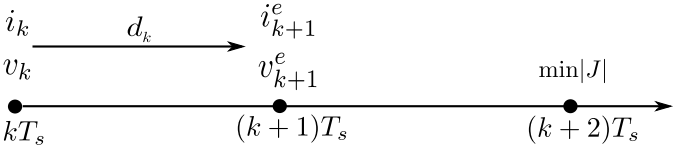


Fig. 3: Proposed CCS-MPC algorithm with computation delay compensation

III. Stability Analysis

To simplify the stability analysis, the effect of computation time delay is ignored. If the SDM is accurate, then the effect of computation delay is compensated accurately, which is as good as having no computation delay. If the system is stable, then in a steady state, the system settles to the steady state value of state variables v^* and i^*. This operating point in the sampled data model is expressed as follows:

$$i^* = Ai^* + Bv^* + E(d^*)V_g \tag{4}$$
$$v^* = Ci^* + Dv^* + F(d^*)V_g \tag{5}$$

considering a perturbation around the steady state point i^*, v^* at the start of k^{th} cycle, for the proposed control algorithm, the state variables will be perturbed by a certain amount as:

$$i[k] = i^* + \tilde{i_k},\ v[k] = v^* + \tilde{v_k} \tag{6}$$

Minimizing the cost function in (3) leads to the following relation between the perturbations in voltage and current:

$$J = \{(\alpha(v^* - v^* + \tilde{v_k}) + (1-\alpha)(i^* - i^* + \tilde{i_k})\}^2 \tag{7}$$
$$J = 0 \Rightarrow \tilde{v_k}/\tilde{i_k} = -(1-\alpha)/\alpha \tag{8}$$

This is also carried forward in subsequent cycles; for the next cycle k+1, the control algorithm computes optimal duty ratio d_k to take J to 0 with different perturbations in voltage and current maintaining the same ratio.

$$\frac{\tilde{v_k}}{\tilde{i_k}} = \frac{\tilde{v_{k+1}}}{\tilde{i_{k+1}}} = \frac{-(1-\alpha)}{\alpha} = \beta \tag{9}$$

From sampled data model,

$$i[k+1] = i^* + \tilde{i_{k+1}} = Ai[k] + Bv[k] + E(d_k)V_g \tag{10}$$
$$v[k+1] = v^* + \tilde{v_{k+1}} = Ci[k] + Dv[k] + F(d_k)V_g \tag{11}$$

Substituting (4) and (5) in (10) and (11)

$$\tilde{i_{k+1}} = A\tilde{i_k} + B\tilde{v_k} + (E(d_k) - E(d^*))V_g \tag{12}$$
$$\tilde{v_{k+1}} = C\tilde{i_k} + D\tilde{v_k} + (F(d_k) - F(d^*))V_g \tag{13}$$

When the perturbation is small, d_k is close to d^*; therefore the following approximations can be made:

$$E(d_k) - E(d^*) \approx \frac{dE}{dd}(d_k - d^*) \tag{14}$$
$$F(d_k) - F(d^*) \approx \frac{dF}{dd}(d_k - d^*) \tag{15}$$

Substituting (14) and (15) in (12) and (13), respectively, while also considering (9)

$$\tilde{i_{k+1}} \approx A\tilde{i_k} + B\beta\tilde{i_k} + \left(\frac{dE}{dd}\right)(d_k - d^*)V_g \tag{16}$$
$$\beta\tilde{i_{k+1}} \approx C\tilde{i_k} + D\beta\tilde{i_k} + \left(\frac{dF}{dd}\right)(d_k - d^*)V_g \tag{17}$$
$$\Rightarrow \frac{\tilde{i_{k+1}} - A\tilde{i_k} - B\beta\tilde{i_k}}{dE/dd} \approx \frac{\beta\tilde{i_{k+1}} - C\tilde{i_k} - D\beta\tilde{i_k}}{dF/dd} \tag{18}$$
$$\frac{\tilde{i_{k+1}}}{\tilde{i_k}} \approx \frac{AdF/dd + \beta BdF/dd - CdE/dd - \beta DdE/dd}{dF/dd - \beta dE/dd} \tag{19}$$

The ratio of $\tilde{i_{k+1}}$ to $\tilde{i_k}$ which is a measure of steady-state stability, is plotted as a function of steady-state duty ratio for different α in Fig. 4. If the absolute value of this ratio is maintained below one, then the perturbation in the inductor current and output voltage settles to zero in a steady state. It can be seen that there is a critical duty ratio for some alpha above which the ratio is less than -1. These regions are unstable. The solution of equating (19) to -1 gives the critical

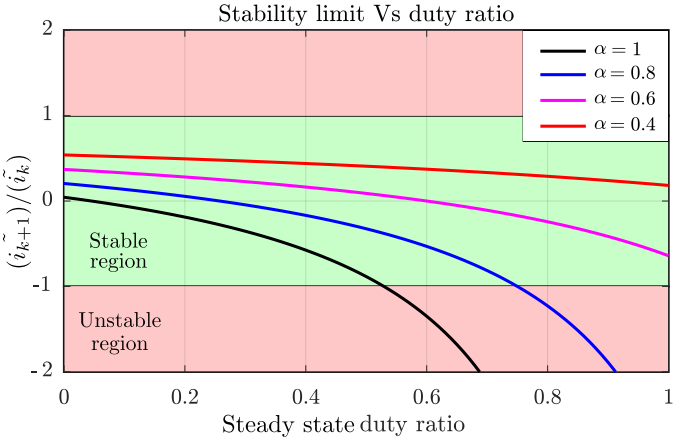


Fig. 4: Stability analysis of proposed algorithm

duty ratio, $d_{crit}(\alpha)$ below which the algorithm is stable.

$$\frac{(A+\beta B)dF/dd - (C+\beta D)dE/dd}{dF/dd - \beta dE/dd} = -1 \tag{20}$$
$$dF/dd(A + \beta B + 1) - dE/dd(\beta + C + \beta D) = 0 \tag{21}$$

Equation (21) is a transcendental equation closed for expression cannot be obtained for the exact sampled data model involving exponential and trigonometric function. An approximate expression for the critical duty ratio can be obtained with polynomial expression for exponential and sinusoidal functions. This is valid for most practically designed buck converter.

$$d_{crit} \approx 1 - \frac{2\beta + 2R\zeta + 2\beta\zeta^2\omega^2 - 2R\zeta^2\omega - \beta\omega^2/2 - 2\beta\zeta\omega}{4R\zeta\omega + \beta\zeta\omega^3 - R\zeta\omega^3 - \beta\omega^2} \tag{22}$$

The critical duty ratio from (22) is plotted as a function of alpha in Fig. 5 which makes the algorithm stable. The plot

is valid for the converter parameters given in Table. I as the critical duty ratio is a function of T_s (switching time period), L (filter inductance), C (filter capacitance), and R (load resistance). Based on this α is chosen as 0.6 considering safe margin for stability.

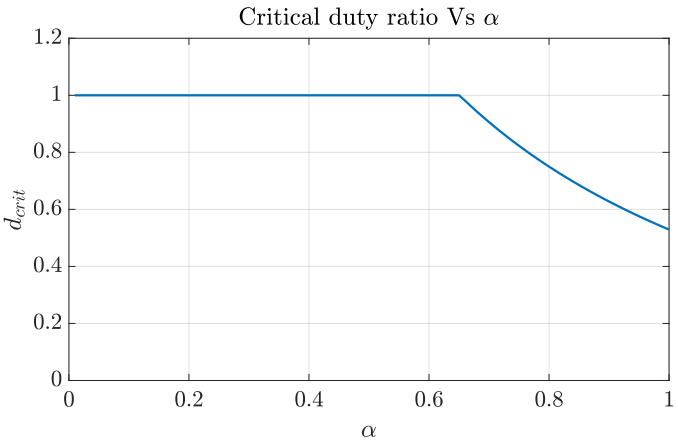


Fig. 5: Critical duty ratio as a function of α

IV. Simulation and Experimental Results

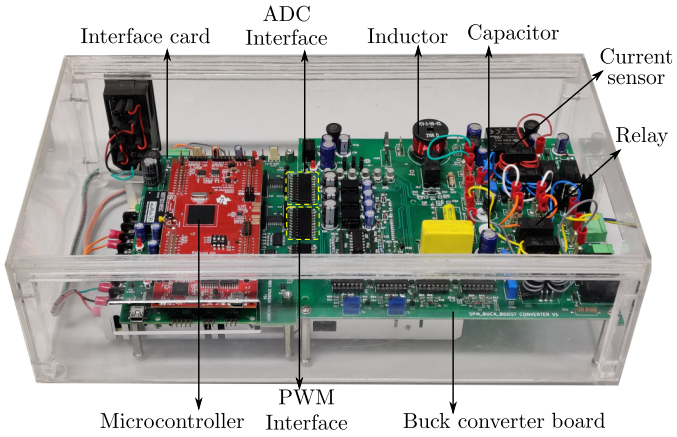
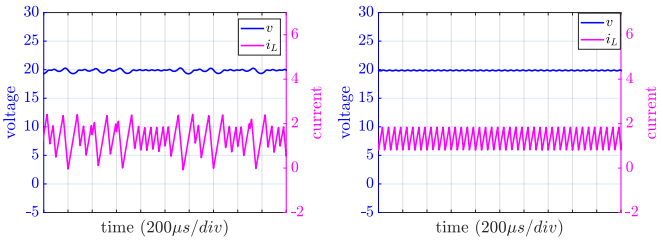


Fig. 6: Hardware setup for the buck converter

TABLE I: Converter Parameters

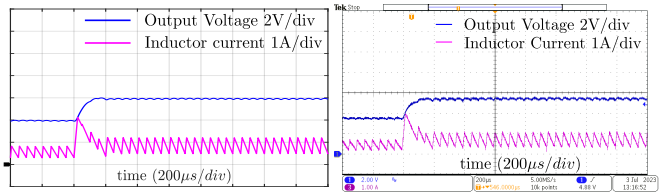
Parameter	Symbol	Value
Input Voltage	(V_g)	30 V
Filter Inductance	(L)	330 μH
Filter Capacitor	(C)	47 μF
Load Resistance	(R)	$4 - 15\ \Omega$
Switching frequency	(f_s)	20 kHz
Update period	(T_s)	50 μs

The simulation is carried out in the simulation software MATLAB Simulink, and the results are given in this section. The parameters for the simulation and experiment are given in Table.I. The steady state oscillation observed for a 20V reference in dead beat voltage control buck converter is shown in Fig. 7a. This oscillations are eliminated in proposed strategy as shown in Fig. 7b, which improves the steady state response.



(a) Dead beat voltage control (b) Proposed MPC

Fig. 7: Steady state response for 20V reference ($V_g = 30V$)



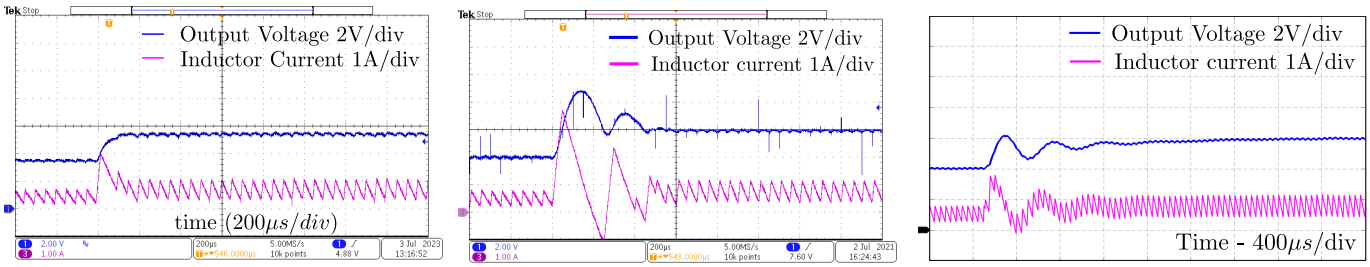
(a) Simulation (b) Experimental

Fig. 8: Response for a step change in reference from 4V to 6V for the proposed strategy

Response for step change in reference from 4V to 6V is shown in Fig 8 with simulation and experimental results. It can be seen that the reference is tracked within 4-5 switching cycles without any overshoot for the proposed MPC strategy. The dynamic performance is compared with dead beat voltage control and classical PI with lead control as given in Fig. 9.The dynamic response is superior compared to dead beat voltage control and classical frequency domain controller while maintaining the steady state performance. The transient performance is compared with dead beat voltage control for a reference change from 12V to 18V, 8V to 10V and load change from 15Ω to 4Ω. It can be observed from the Fig. 10 that the proposed strategy has better transient and steady state performance compared to dead beat voltage control. The experimental setup for the buck converter is shown in Fig. 6.

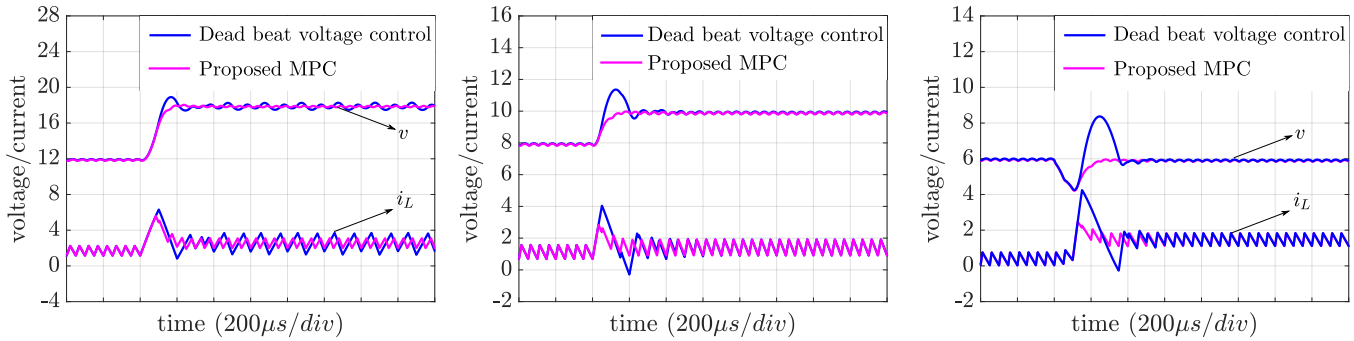
V. Conclusion

The paper proposes a novel CCS-MPC algorithm for a buck converter considering both the state variables in the cost function. The proposed algorithms has a superior transient performance compared to existing solutions. It uses a modified cost function considering both the state variable inductor current and output voltage. The necessity for considering inductor current in the cost function is highlighted with stability analysis and is considered with a weighting factor. Stability was analyzed for variation in weighing factor, and an analytical expression for the stability limit involving the critical duty ratio was given. The modified algorithm improves the transient and steady state response compared to dead beat



(a) Proposed Model Predictive Control (b) Dead beat voltage control (c) PI with lead controller

Fig. 9: Comparison of response for a step change in reference from 4V to 6V



(a) Step change in ref from 12V to 18V (b) Step change in ref from 8V to 10V (c) Load change from 15Ω to 4Ω

Fig. 10: Comparison of transient and steady state performance with dead beat voltage control for buck converter.

voltage control. The algorithm was verified in simulation and implemented in microcontroller with reduced computational complexity.

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